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PDCAT Tutorial

Cooperative Mobile Robots
System Models & Basic Terminology

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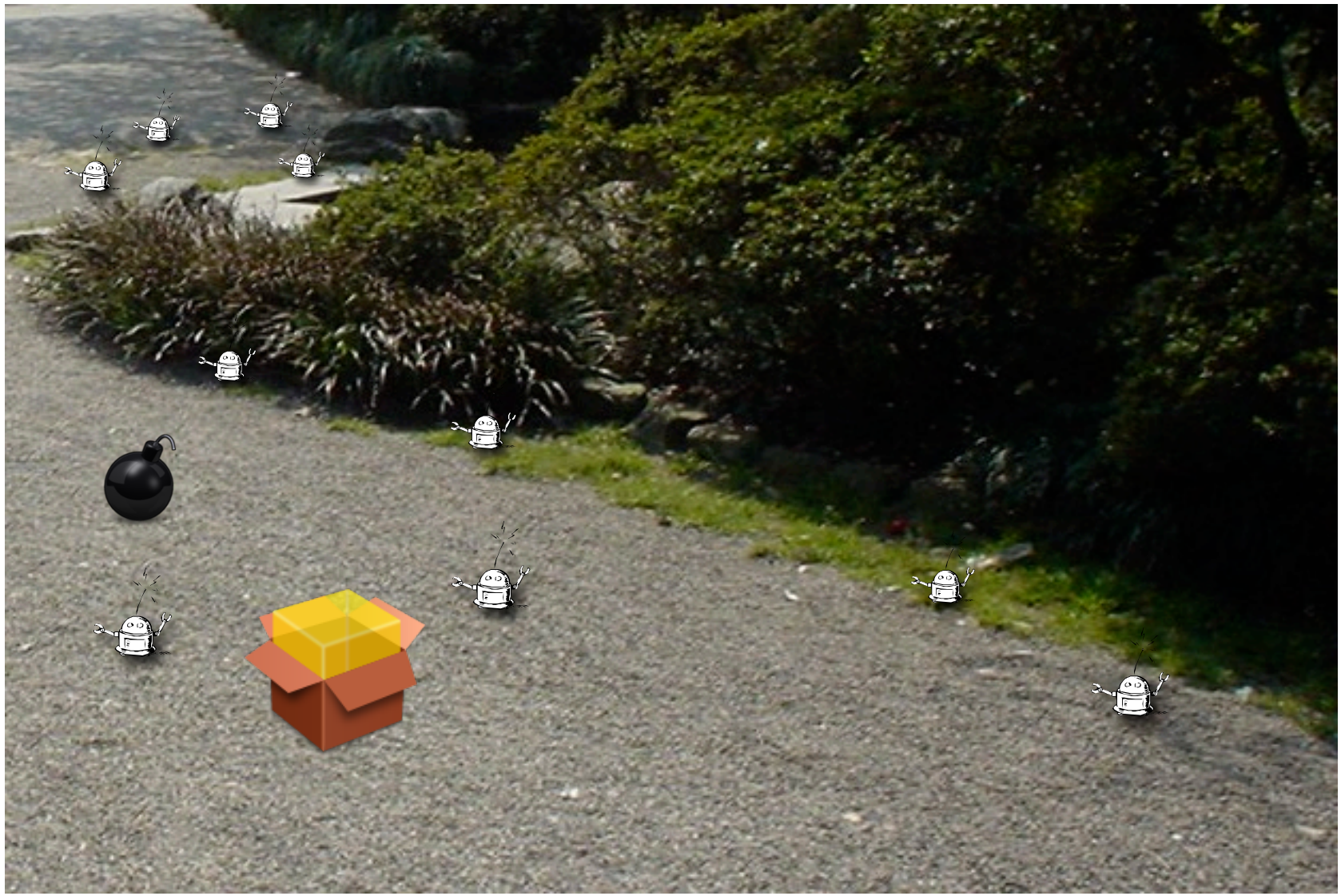
Simple Illustration



Context & Motivation

- **Bottom-Up**
  - 1./ Build robots
  - 2./ Enforce coordination
  - => Realistic
  - => Coordination is an afterthought
- **Emergence / Stigmergy**
  - 1./ Define simple behavior
  - 2./ Study combined behavior
  - 3./ Try to infer rules
  - => Works well ... on average cases

Context & Motivation

- **Algorithmic Approach**
  - 1./ Define model & problem
  - 2./ Find minimal set of assumptions
  - 3./ Prove possibility / impossibility
  - => Fundamental limits of coordination

- **Highlights**
  - Top-down approach
  - Aims for self-organizing properties
  - Combines:
    - Distributed algorithms, computational geometry

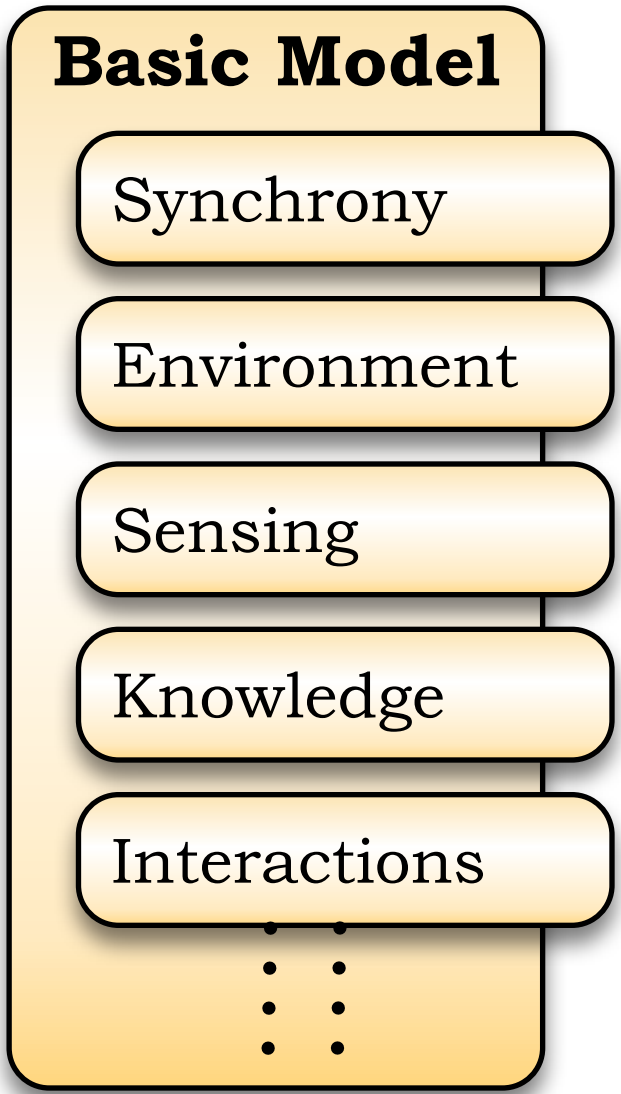
Outline

- **System Models**
  - Basic Model
- **Model Variants**
  - Synchrony, etc...
- **Main Problems**
  - Definitions
  - Some Known Results
- **Leads for Discussion**

System Models

System Overview

- **Environment**
  - 2D Euclidean plane
  - No landmarks / obstacles
- **Robots**
  - Process + Location
  - Private coordinate system
  - LOOK - COMPUTE - MOVE
- **Interactions**
  - Observations, no comm.
  - Activations



Model

Original Model

[SY99] Suzuki, Yamashita. Distributed Anonymous Mobile Robots: formation of geometric patterns. SIAM J. Comp., 28(4):1347–1363 (1999).

- Called: **SYm**, SSYNC, ATOM.
- Starting point for other models/variants

SSYNC ATOM

Environment

- **2D Euclidean Space**
  - Continuous coordinates
  - Euclidean distance
  - No boundaries
  - No landmarks
  - No obstacles

  } similar w.r.t. symmetry

$$Z = (o, d, u)$$

Environment

- **Coordinate system**

$$Z = (O, \vec{x}, \vec{y})$$

  - Origin, direction of x-axis, unit distance
  - Global system is **unknown** to robots

- **Time**
  - Discrete time: 0,1,2,...
  - Time is **unknown** to robots

Robots

- **Robots in the System**

$$\{r_1, r_2, \ldots, r_n\}$$

  - Robots are dimensionless (i.e., points)
  - Can move
  - Possibly occupy the same location
  - Are undistinguishable
  - Execute the same code
  - Do not know their identifier (anonymous)
    - Disallows:

```
if me == r_x then ...
```

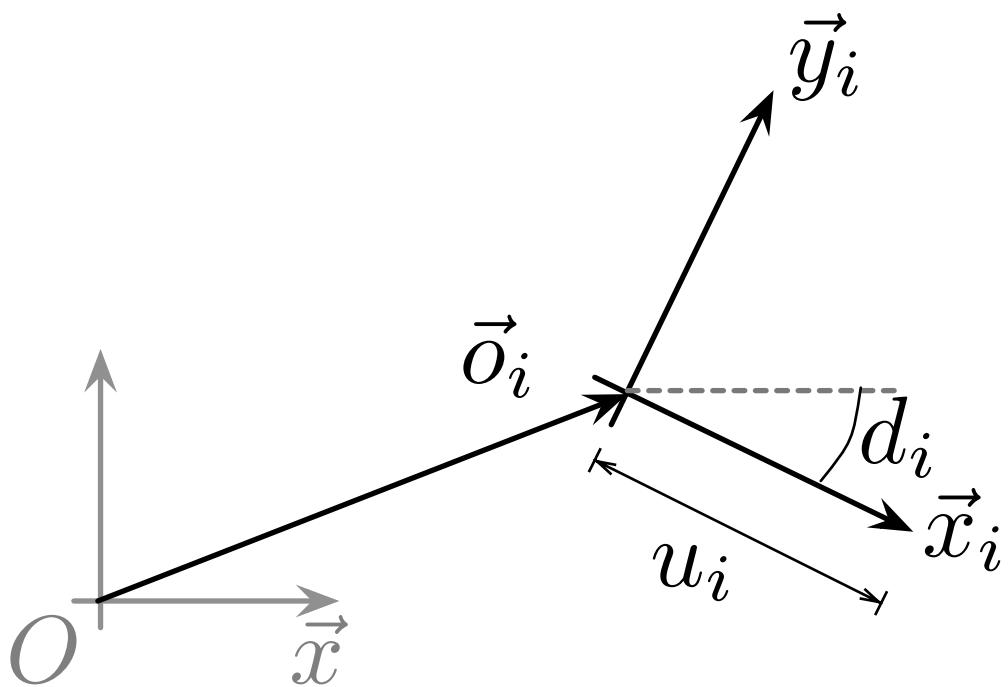
Robots

Let's look at robot r_i

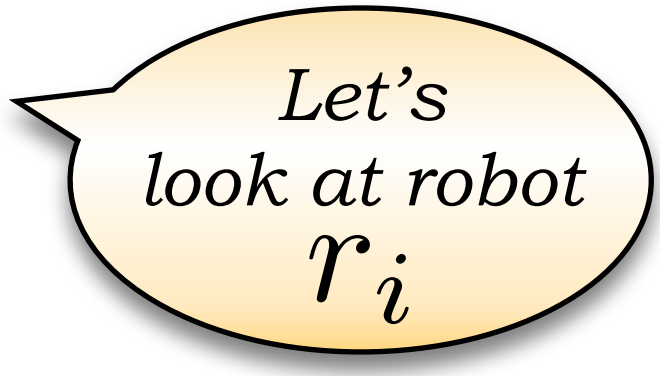
Local Coordinates

$$Z_i = (\vec{o}_i, d_i, u_i)$$

- Common sense of orientation (chirality):
  - x,y-axes at 90° counter-clockwise



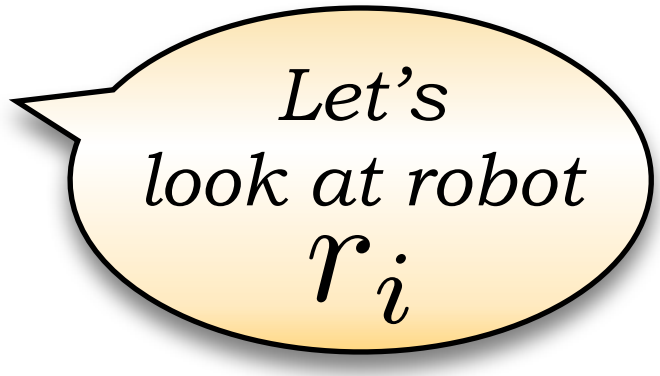
Robots



- **Position**
  - $p_i(t)$: position of robot r_i at time t
- **Initial Position**
  - $p_i(0)$
- **Positions**
  - $P(t) = \{p_i(t) | 1 \leq i \leq n\}$
  - Multiset => several robots @ same position
  - Position distinct initially,

$$\forall i, j : i \neq j \Rightarrow p_i(0) \neq p_j(0)$$

Robots



- **Algorithm**
  - Sequential process
  - Robot is: *active* | *inactive*
  - Activated randomly
- **Activation**
  - **LOOK** at the environment
  - **COMPUTE** a target destination
  - **MOVE** toward the target

LOOK

Let's look at robot r_i

Notation

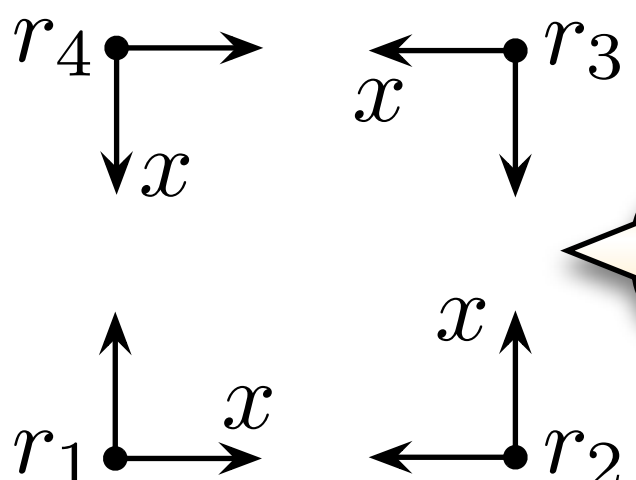
- $[p]_i \equiv p$ in terms of Z_i

Observation

- Get a set of points

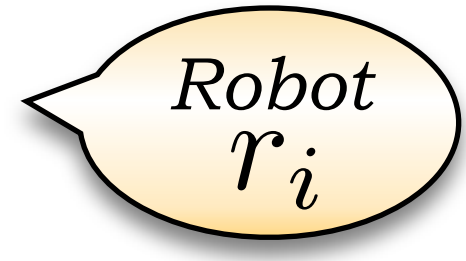
$$[P(t)]_i = \{[p_k(t)]_i \,|1 \leq k \leq n\}$$

- Get its own position



symmetrical

COMPUTE



- **Compute Target Destination**
  - Outputs a single point

$$\psi([P(t)]_i\,,[p_i]_i) \mapsto [p'_i(t)]_i$$

  - Function is deterministic
  - Computes target destination
- **Variants:**
  - **Oblivious**: function is stateless
  - **Non-oblivious**: retains past observations

MOVE

- **Move Toward Target**
  - Move toward target destination
  - **Limited** movement
    - Robot has reachability $\Delta_i > 0$ *(unit distance)*
    - Moves to point within Δ_i nearest to target
    - $\Delta_i > 0$ is not infinitesimally small
  - New position reached before time $t + 1$

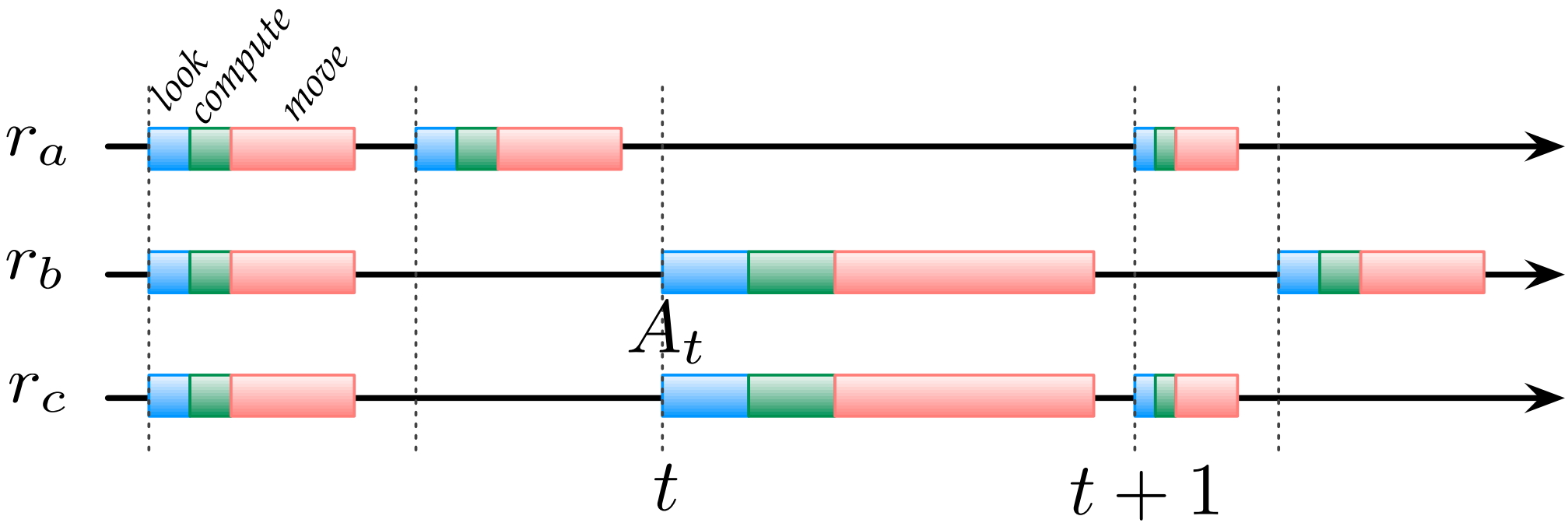
Activation Schedule

- **Activation**
  - Infinite sequence: $\mathcal{A} = A_0, A_1, \ldots, A_t, \ldots$
  - A_t : set of active robots at t
- **Conditions**
  - At each time:

    at least one robot is active
  - For every robot:

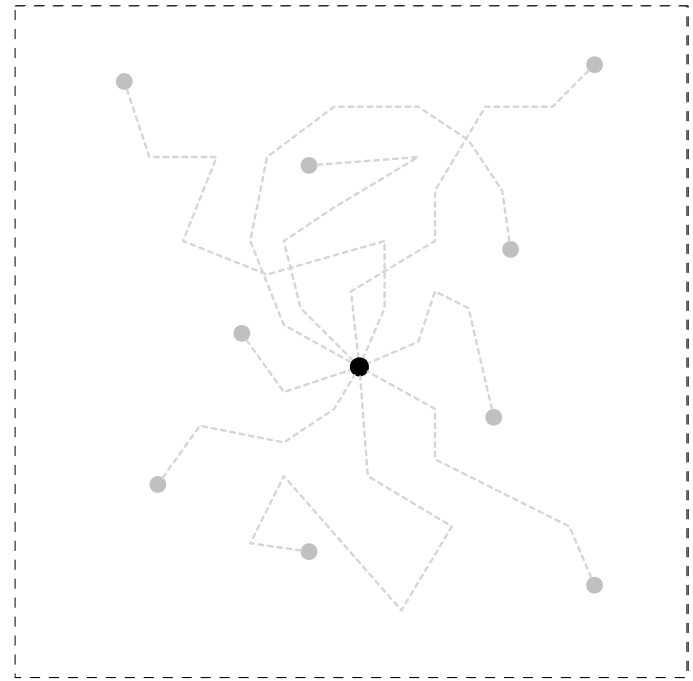
    it is active infinitely-many times
- **Active Robot**
  - Executes: LOOK - COMPUTE - MOVE

Activation Schedule



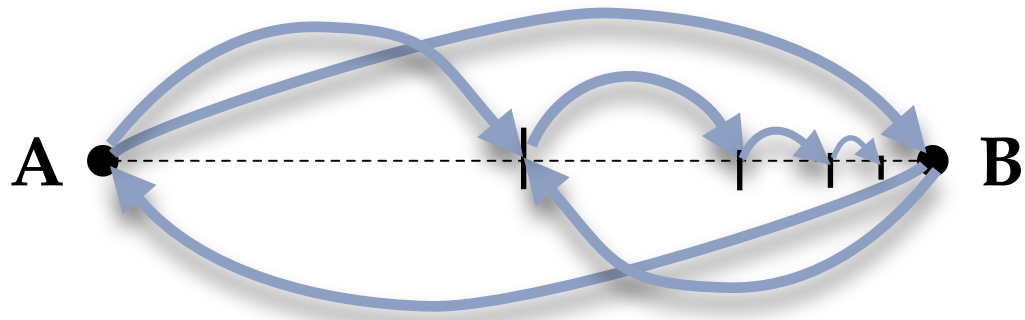
- **Special Cases**
  - Synchronized: $\forall t : \|A_t\| = n$
  - Centralized: $\forall t : \|A_t\| = 1$

Gathering Problem



- **Initially**
  - Robots located arbitrarily
- **Objective**
  - Eventually all robots at same location
- **Formation**
  - Must reach goal in finite number of steps
  -

Difficulty of Gathering



- **Illustration**
  - Two robots **A** and **B**
  - SYm model
  - Deterministic algorithm
  - **Oblivious** robots

    => 2-Gathering impossible [SY99]

Model Variants

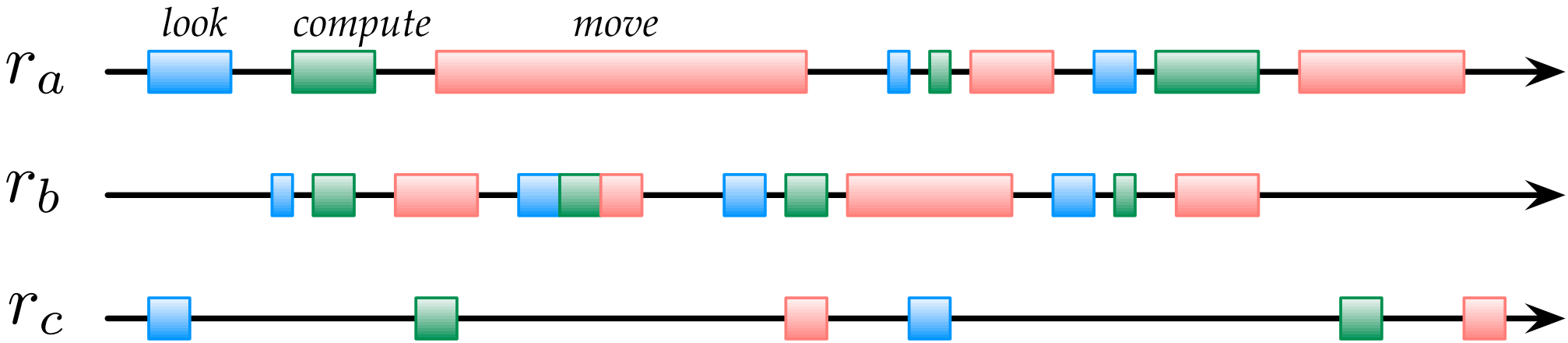
Activation Schedule

- **Original Model**
  - Called: **SYm**, SSYNC, ATOM
- **Asynchronous**

[FPSW05] Flocchini, Prencipe, Santoro, Widmayer. Gathering of asynchronous robots with limited visibility. T.C.S., 337:147–168, 2005.

  - Called: **CORDA**, ASYNC
  - Separate events:
    - LOOK
    - COMPUTE
    - BEGIN MOVE
    - END MOVE

CORDA Model



- **Important Points**
  - Robots can be seen **while moving**
  - Different robot "speed"
- **Special Cases**
  - SYm strictly included in CORDA

Multiplicity

- **No multiplicity detection**
  - Set of points
  - Two+ collocated robots seen as single point.
- **Multiplicity detection**
  - Point of multiplicity identified ***(weak)***
  - Count of collocated robots ***(strong)***

Visibility

- **Full visibility**
  - All robots included in the set $P(t)$
- **Limited visibility**
  - Visibility range R
  - Robot r_i gets subset:
    - Only the robots within distance R
  - Usually:
    - Initially, visibility graph is connected

Memory

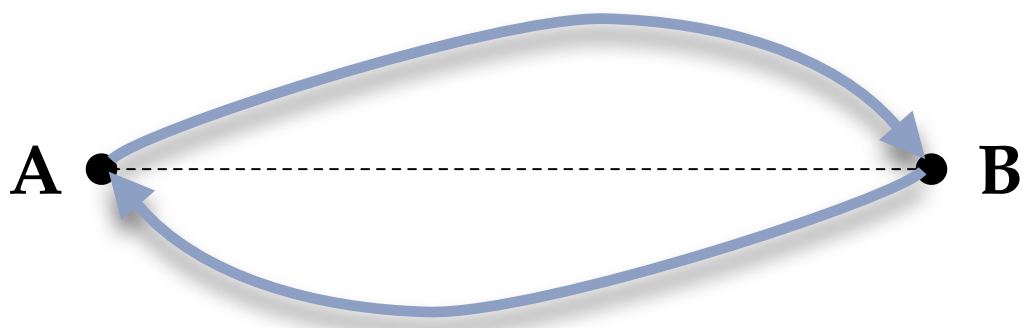
- **Non-oblivious**
  - Input includes all / part of past activations
  - Activation may include stateful information
- **Oblivious**
  - Input is last observation only
  - Activation is stateless

Identity

- **Anonymous**
  - Robots have no identity information
- **With Identity**
  - Some robot can be designated as “leader”
  - Allows: `if me == “leader” then ...`
  - NB: Identity is visible or not

Determinism

- **Deterministic**
  - Algorithm allows only deterministic operations
- **Randomized**
  - Algorithm allows probabilistic operations



Volume

- **Dimensionless Robots**
  - Robots can share the same location
  - Robots have no size
- **Robots with Volume**
  - Robots have a size
  - Robots should not collide

Movement

- **Instantaneous**
  - Robot “teleports” to the target
- **Duration / Path**
  - Movement takes some time
  - Robots follows a path
  - Additional possibility of collision
- **Note**
  - Nearly identical for SYm model

Compass

- **No assumption**
  - Original SYm model
- **Compass**
  - Robots agree on a common direction
  - No agreement on origin
- **Eventual Compass**
  - Robots eventually agree on a common dir.
- **Faulty Compasses**
  - Robots agree within some angle error
  - Static / dynamic variants

Schedulers

- **Fairness**
  - Unfair: may active some robots unfairly
  - Fair: all robots activated infinitely-often
- **k-Bounded**
  - At most *k* activations of any robot *r'* between 2 activations of some robot r
- **Centralized**
  - At most 1 robot active

Fault-Tolerance

- **Crash**
  - Some robots may crash:
    - Disappear from the system (trivial)
    - Stop moving
- **Byzantine**
  - Some robots move against the algorithm
  - Strong/weaker than scheduler
  - Aware/unaware of scheduler decision
  - Aware/unaware of algorithm decisions

Error-Tolerance

- **Errors**
  - Errors on observations
  - Usually:
    - Errors are relative wr.t. robot's location
    - Errors are bounded
    - Bounds are known

Problems

Gathering / Scattering

- **Gathering**
  - Initially: arbitrary configuration
  - => all robots share same location
    - permanently
    - location is not predetermined
- **Scattering**
  - Initially: robots possibly at same location
  - => all robots have distinct positions
    - permanently
- **NB: Dual problems**

Pattern Formation

- **General Pattern**
  - Initially: arbitrary configuration
  - Given: geometric shape
  - => Organize into the shape
- **Special Cases**
  - Circle formation (regular n-gon)
  - Point formation (gathering)
- **Related**
  - Leader election

Flocking

- **Flocking**
  - Initially: arbitrary configuration
  - => Robots eventually move together
- **Variants**
  - Move while keeping **given shape**
  - Move while following **given path**
  - Assumes leader or not
  - …

Discussions

- **Summary**
  - SYm model, CORDA model
  - Many model variants
  - Many fundamental problems
- **Approach**
  - Formal model
  - Proofs of correctness
- **Aim**
  - Minimum requirements for cooperation

Important References

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