Leader Election and Pattern Formation in Swarms of Deterministic Robots

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Swarm of robots

Set of independent, autonomous, and mobile robots

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→ **Characteristics**

- Very little
- Very simple
- Very limited capacities:
  - Power resource
  - Processing
  - Memory
  - Anonymity
  - No means of (direct) communication

Swarm of robots

Set of independent, autonomous, and mobile robots

→ **Applications**

- Impossible tasks for a single robot
- Fault/Destruction tolerance, Dynamicity
- Examples:
  - Risky area surrounding or surveillance
  - Search and rescue missions
  - Exploration of awkward environments
  - Large-scale construction
  - Environmental monitoring
  - Space exploration
  - Military operations

Swarm of robots

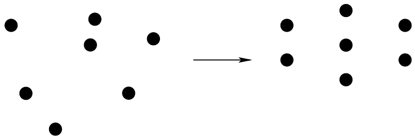
Set of independent, autonomous, and mobile robots

→ **Specific Tasks**

- Movement Management
  - Movement limitation (Performance)
  - Collision avoidance
  - 2D/3D settings
- Distributed control
  - No central coordination
  - Scalability
  - Dynamicity
- Cooperative Tasks

Cooperative Tasks

- Pattern Formation
- Leader Election
- Gathering
- Scatter
- Deployment



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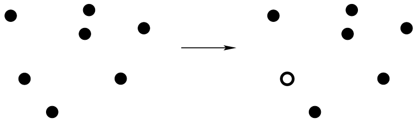

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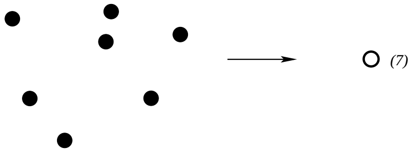

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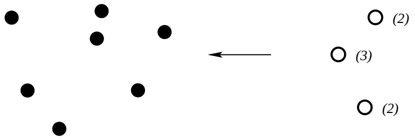

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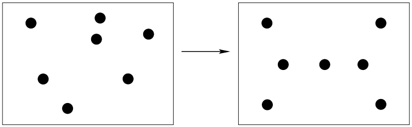

Cooperative Tasks

→ **Approaches**

- Emergent Behavior
  - Artificial Intelligence
  - Behavior of Social Animals (Insects)
  - Approximation methods
- Distributed Algorithms
  - Correctness Proofs

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- Which are the elementary tasks that can be achieved deterministically?
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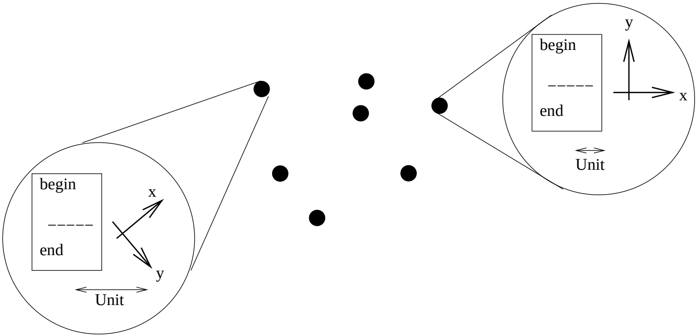
This Talk

- Pattern Formation
- Leader Election
- Gathering
- Scatter
- Localization
- Deployment
- Exploration

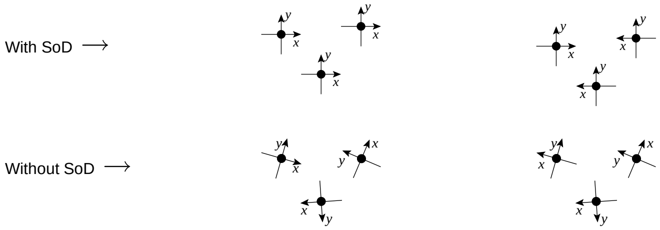
Basic Settings

- Homogeneous
- Anonymous
- Autonomous
- Oblivious
- Moving onto the plan
- Deaf and Dumb
- Unlimited Vision



Orientation Capabilities

- **Sense of Direction**
  The robots agree on a common direction and orientation, (e.g., *North*=*y*)
- **Chirality**
  the robots share the same *handedness*, *i.e.*, the orientation of the *y*-axis is inferred *w.r.t.* the *x*-axis



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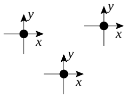
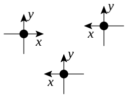
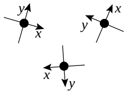


Both SoD and Chirality



SoD without Chirality



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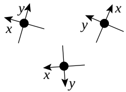


No SoD nor Chirality

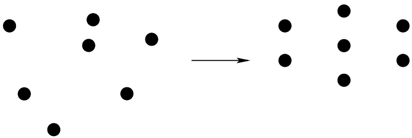
Computation

SYm [Suzuki Yamashita 96]

- An inifinite sequence of time instants $\{t_0, t_1, \ldots, t_i, \ldots\}$
- At each time instant t_i, each robot is either active or idle
- In t_i, every active robot executes the following phases:
  1. Observe all the positions
  2. Compute a destination d
  3. Move toward d

     Note: The distance traveled in 1 step by any robot r is bounded by σ_r.

Problem



Definition (Arbitrary Pattern Formation)

Given a swarm of n robots scattered on the plan, designing a deterministic algorithm so that, the robots eventually form a pattern $\mathcal{P}$ made of n positions and known by each of them in advance.

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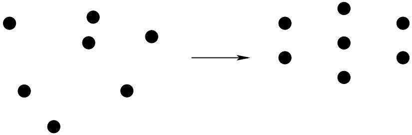

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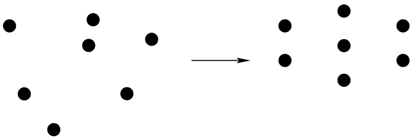

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Given a swarm of n robots scattered on the plan, designing a deterministic algorithm so that, the robots eventually form a pattern $\mathcal{P}$ made of n positions and known by each of them in advance.

In other words, at the end of the computation, the positions of the robots coincide, with the positions of $\mathcal{P}$, where $\mathcal{P}$ can be translated, rotated, and scaled in each local coordinate system.

Previous Works

SoD and Chirality

[Flocchini et al., 2001]

A solution exists for any n and any $\mathcal{P}$.

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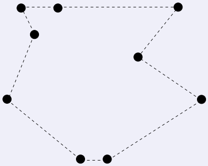

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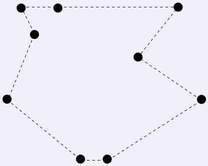

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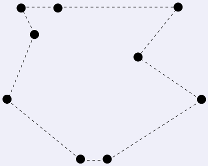

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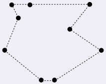

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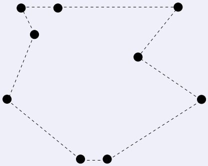

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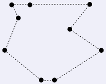
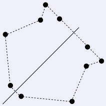


If n is even, a solution exists provided that $\mathcal{P}$ is symmetric.



Pattern Formation With No Sense of Direction

Question

Assuming no sense of direction (with or without chirality), which kind of patterns can be formed in a deterministic way?

Pattern Formation With No Sense of Direction

Theorem

[Flocchini et al., 2001]

If the robots share no Sense of Direction, then they cannot solve the Arbitrary Pattern Formation problem deterministically, even having the ability of chirality.

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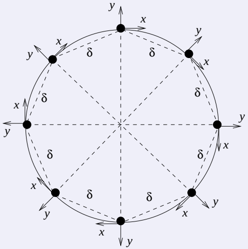

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Let Θ be the class of patterns that can be solved deterministically assuming robots devoid of sense of direction.

Corollary

Either Θ *is equal to the set of regular polygons (n-gons)*
or $\Theta = \emptyset$.

Circle Formation

Question

Assuming no sense of direction, is it possible to eventually form a regular n-gon (circle)?

Circle Formation

Previous work

[Défago and Konagaya, 2002] [Chatzigiannakis et al., 2994] [Défago and Souissi, 2008]

Asymptotic convergence toward the n-gon.

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Any n (in *CORDA*), but the n-gon is not systematically achieved.

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1. Move the robots on (the boundary of) a circle $\mathcal{C}$.
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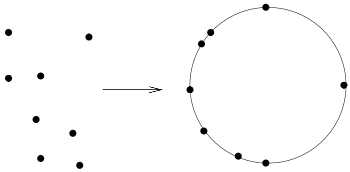

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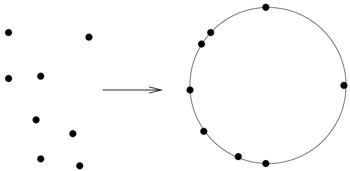

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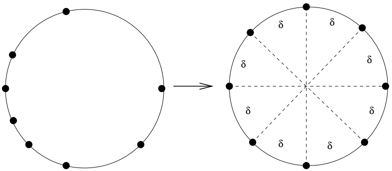

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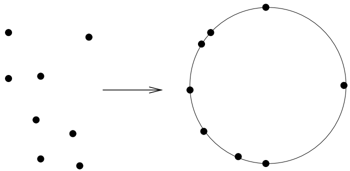

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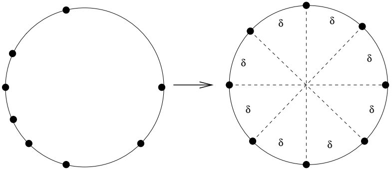

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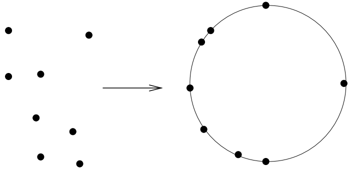

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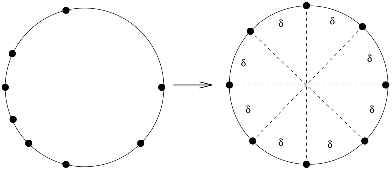

HARD PART

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Theorem

[Flocchini et al, 2006]

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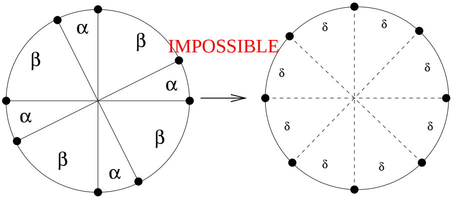

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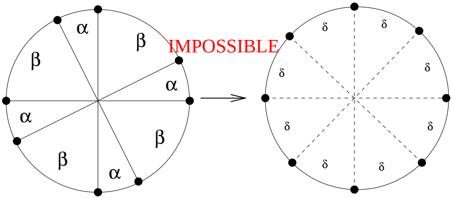

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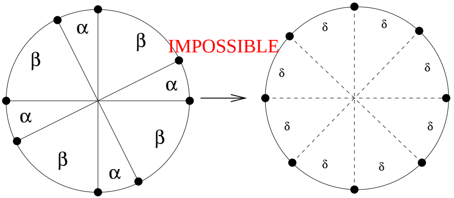

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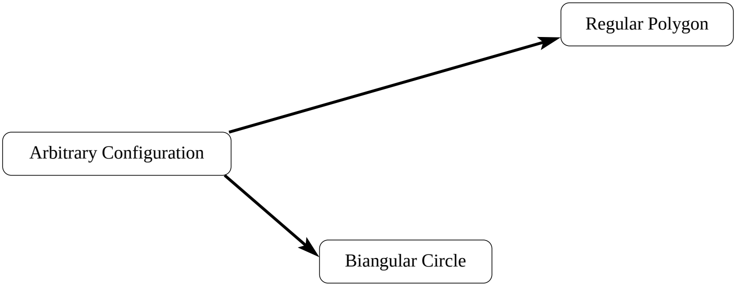
Fortunately, not!

An almost *n*-gon algorithm



[Katreniak, 2005]

Starting from a biangular circle

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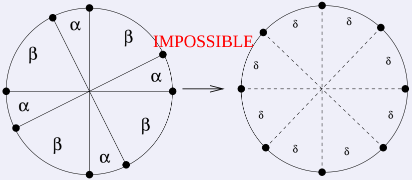
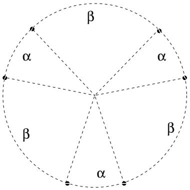


Starting from a biangular circle

Let us leave $\mathcal{C}$



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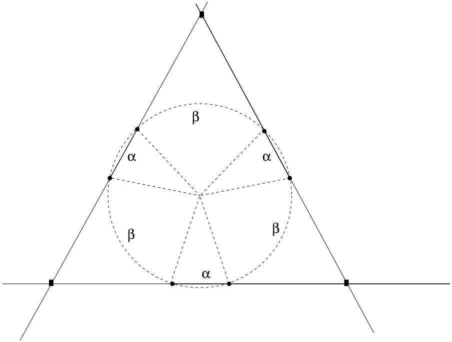

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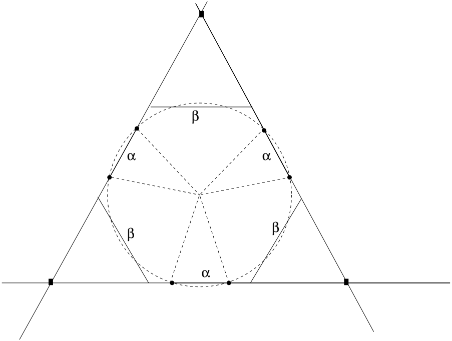

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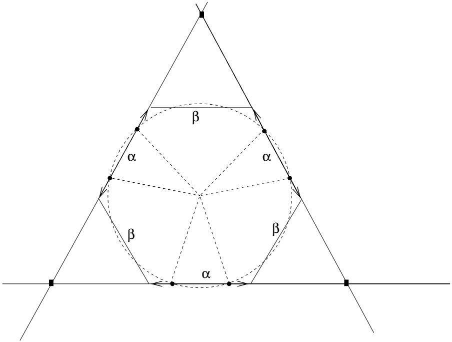

Starting from a biangular circle

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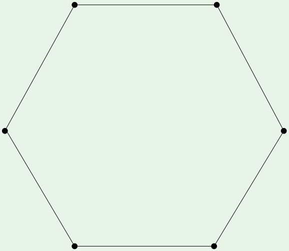
Every robot is active



Regular Polygon

Starting from a biangular circle

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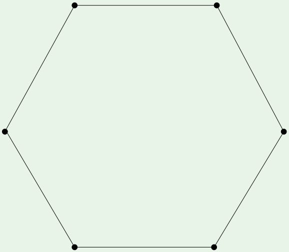
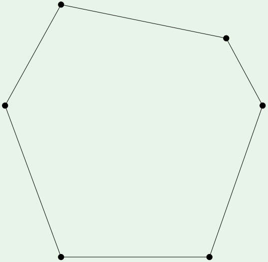


Regular Polygon

Some robots are active



Ideal Polygon

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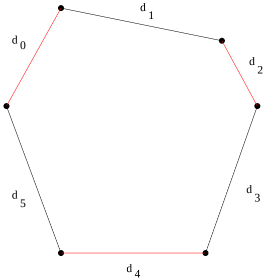

Property 1

- Either $d_0 \geq d_1 \leq \cdots \geq d_{n-1} \leq d_0$
- or $d_0 \leq d_1 \geq \cdots \leq d_{n-1} \geq d_0$

Starting from a biangular circle

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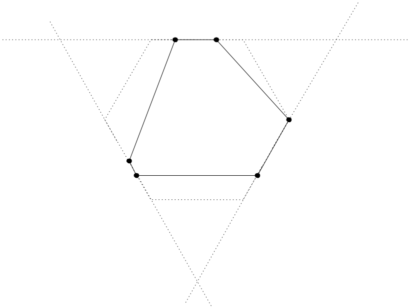

Property 2

A regular n-gon can be associated to an Ideal Polygon.

Starting from a biangular circle

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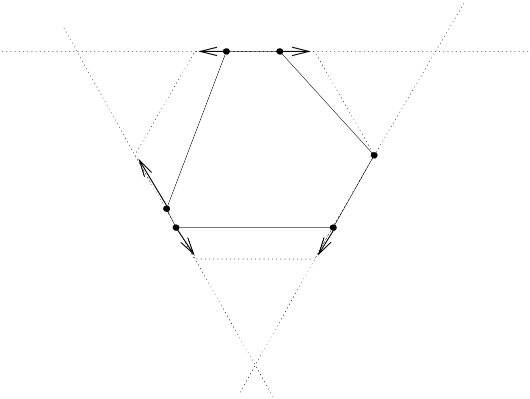

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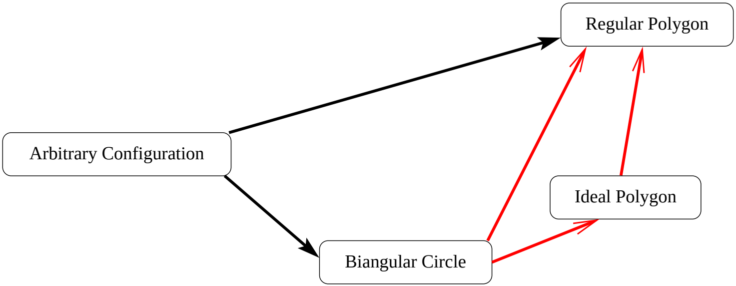
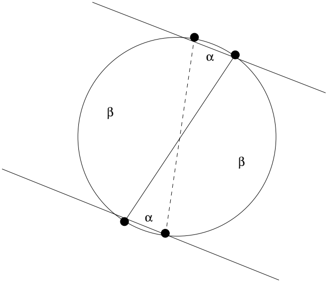
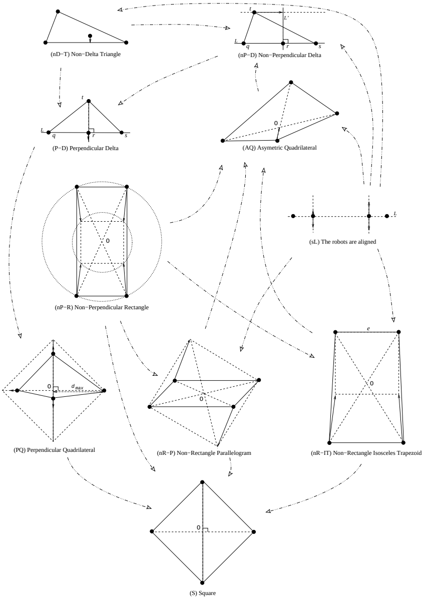


Circle Formation



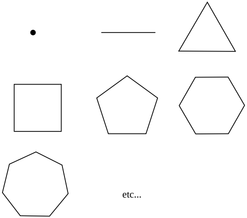
It works if $n \geq 5$ only

1. [Katreniak 2005] works if $n \geq 5$ only
2. Difficulty to find an invariant


(nD−T) Non−Delta Triangle
(nP−D) Non−Perpendicular Delta
(P−D) Perpendicular Delta
(AQ) Asymetric Quadrilateral
(sL) The robots are aligned
(nP−R) Non−Perpendicular Rectangle
(PQ) Perpendicular Quadrilateral
(nR−P) Non−Rectangle Parallelogram
(nR−IT) Non−Rectangle Isosceles Trapezoid
(S) Square

$n \leq 4$

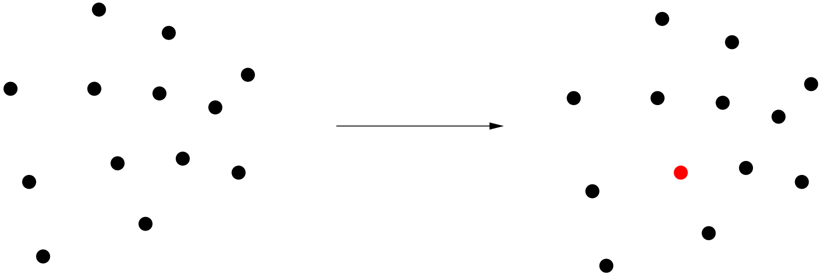
Arbitrary Pattern Formation



Theorem

∀n, the class of patterns that can be solved deterministically *assuming robots devoid of sense of direction (Θ) is equal to the set of regular polygons (n-gons).*

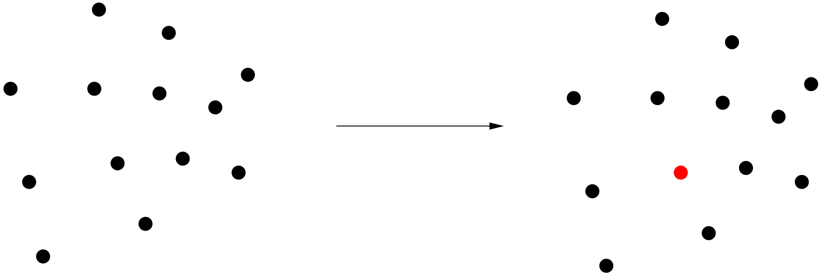
Problem



Question

Given a swarm of n robots, what are the minimal geometric conditions to be able to deterministically agree on a single robot?

Problem



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Previous Works

SoD and Chirality

[Flocchini et al., 1999]

A solution exists for any n.

SoD and No Chirality

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A solution exists if n is odd.

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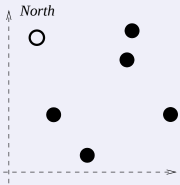

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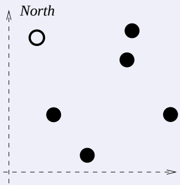

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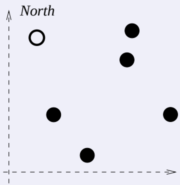

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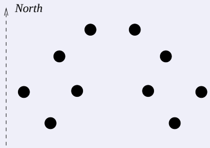

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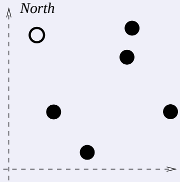

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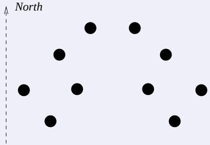

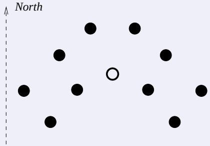

Previous Works

No SoD

[Prencipe 2002]

Impossible in general.

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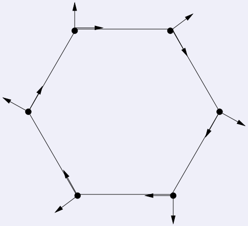

In such a configuration, it is not possible to break the symmetry.

Leader Election With No Sense of Direction

Question

Assuming no sense of direction (with or without chirality), what are the geometric conditions to be able to deterministically agree on a single robot?

To answer to this question, we need tools from the theory on Combinatoric on Words, specifically Lyndon Words.

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To answer to this question, we need tools from the theory on Combinatoric on Words, specifically Lyndon Words.

Lyndon Words

Definition (Word)

Let $A = \{a_0, a_1, \ldots, a_n\}$ be an alphabet. A word is a (possibly empty) sequence of letters in A.

$$A = \{a, b, c, d\}$$

$$abcc \quad a \quad \epsilon \quad dddddddd \equiv d^8$$

Lyndon Words

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Lyndon Words

Definition (Concatenation)

Let $u = a_1, \ldots, a_i, \ldots, a_k$ and $v = b_1, \ldots, b_j, \ldots, b_\ell$.
The concatenation of u and v, denoted uv, is equal to the word $a_1, \ldots, a_i, \ldots, a_k, b_1, \ldots, b_j, \ldots, b_\ell$.

$u = Hiro$, $v = shima$, $uv = Hiroshima$

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Lyndon Words

Definition (Lexicographic Order)

Let A be an alphabet totally ordered by $\prec$, *i.e.*, $a_0 \prec a_1 \prec \ldots \prec a_n$.
A word $u = a_0a_1 \ldots a_s$ is said to be *lexicographically smaller than or equal to a word* $vu = b_0b_1 \ldots b_t$*, denoted by* $u \preceq v$*, iff:*

- *either* u *is a prefix of* v*,*
- *or,* $\exists k : \forall i \in [1, \ldots, k-1], a_i = b_i$ *and* $a_k \prec b_k$.

$ab \preceq abc$ $\quad abc \preceq abc$ $\quad \epsilon \preceq abc$ $\quad abc \preceq def$

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Lyndon Words

Definition (Primitive Word)

A word u is said to be *primitive* iff $u = v^k \Rightarrow k = 1$. Otherwise, u is said to be *periodic*.

Primitive Words

ab *dabcbc* *dcba*

Periodic Words

d^8 *bcbc* ϵ

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Definition (Rotation)

A word u is said to be a *rotation* of a word v iff there exists two words x, y such that $u = xy$ and $v = yx$.

$u = abcd$ and $v = cdab$
$u = abcd$ and $v = bcda$

Definition (Minimality)

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Lyndon Words

Definition (Lyndon Word)

A word u is a *Lyndon word* iff u is primitive and minimal.

Lyndon Word

abc ($abc \preceq cab$ and $abc \preceq bca$)

Not a Lyndon Word

bca ($bca \succ abc$)

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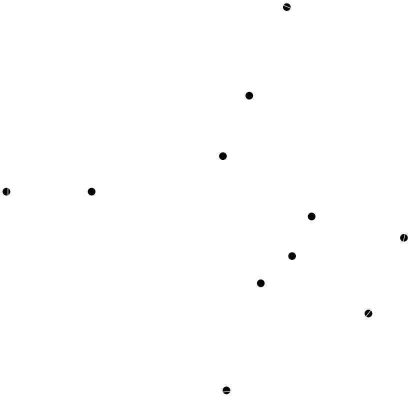
Leader Election with Chirality



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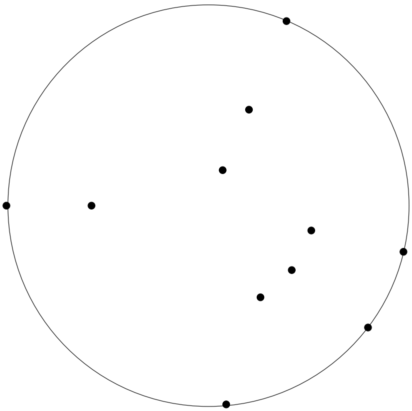

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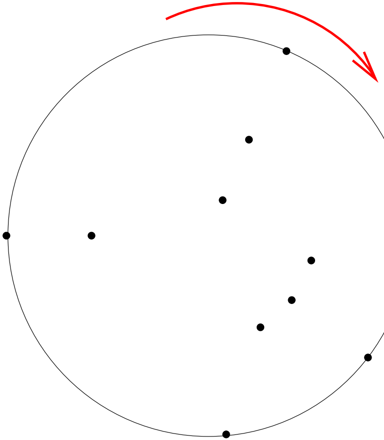

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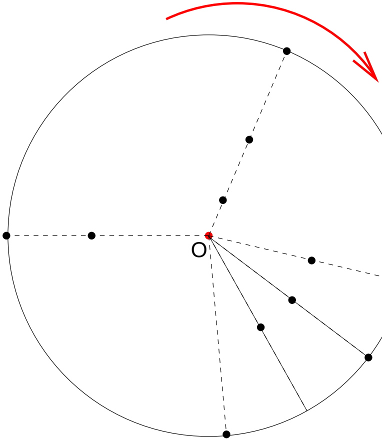

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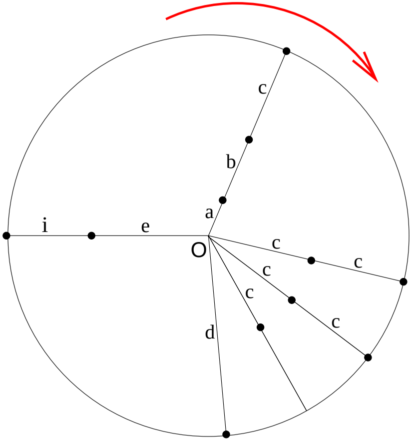

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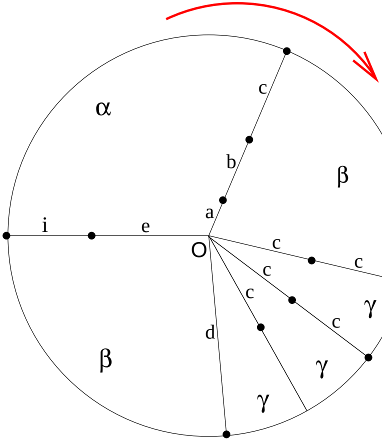

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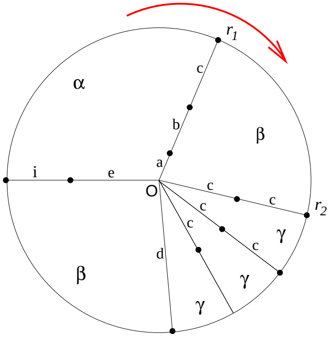


$$W(\rho_1) = (abc, \beta)(c^2, \gamma)^2(c, \gamma)(d, \beta)(ei, \alpha)$$
$$W(\rho_2) = (c^2, \gamma)^2(c, \gamma)(d, \beta)(ei, \alpha)(abc, \beta)$$

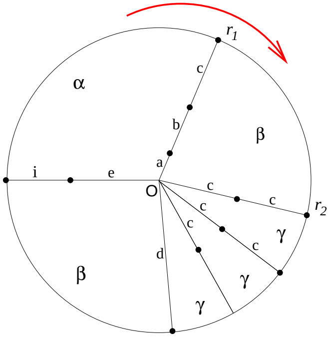
Lemma

If two distinct radii ρ_1 and ρ_2 exist such that both $W(\rho_1)$ and $W(\rho_2)$ are Lyndon words, then $\forall \rho$, $W(\rho) = (0, 0)$.

Lemma ($\Rightarrow$)

If there exists a radius ρ such that $W(\rho)$ is a Lyndon word, then the robots are able to deterministically agree on the same leader.

Leader Election with Chirality



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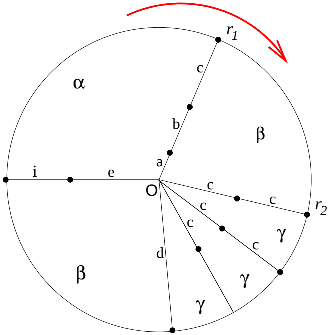

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If there exists a radius ρ such that $W(\rho)$ is a Lyndon word, then the robots are able to deterministically agree on the same leader.

Leader Election with Chirality

Lemma ($\Leftarrow$)

If there exists no radius ρ such that $W(\rho)$ is a Lyndon word, then the robots are not able to deterministically agree on the same leader.

Property

[Lothaire 1983]

If no rotation of a work u is a Lyndon word, then u is periodic.

Leader Election with Chirality

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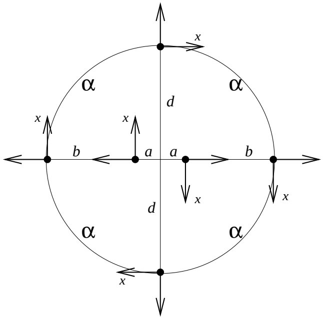


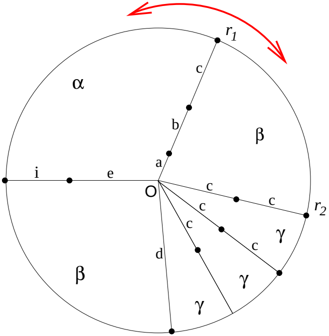
Leader Election with Chirality

Theorem

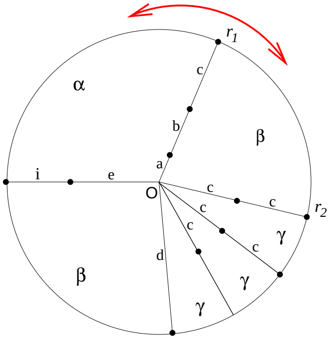
Assuming chirality, a swarm of robots is able to deterministically agree on the same leader if and only if there exists a radius ρ such that $W(\rho)$ is a Lyndon word.

Leader Election without Chirality

Leader Election without Chirality



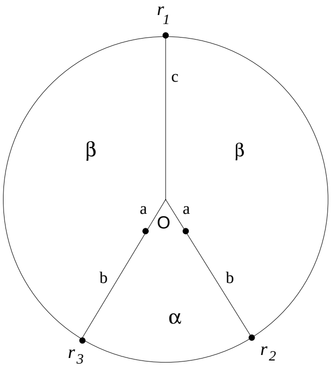
Leader Election without Chirality



For each ρ, there are 2 ways to compute $W(\rho)$

$W(\rho_1) =$
either
$(abc,\beta)(c^2,\gamma)^2(c,\gamma)(d,\beta)(ei,\alpha)$
or
$(abc,\alpha)(ei,\beta)(d,\gamma)(c,\gamma)(cc,\gamma)(cc,\beta)$
depending on either ↻ or ↺, respectively.

Leader Election without Chirality



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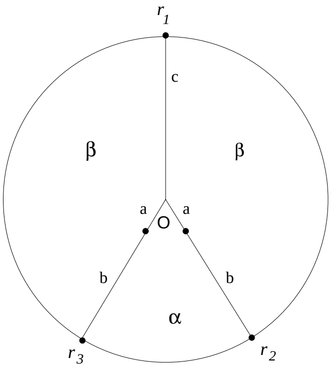
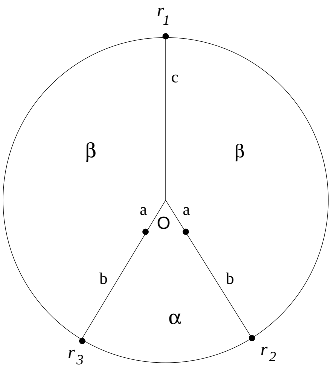


The word $W(\rho_2)^{\circlearrowright} = W(\rho_3)^{\circlearrowleft} = (ab, \alpha)(ab, \beta)(c, \beta)$ is a Lyndon word.

Leader Election without Chirality



Definition (Type of Symmetry)

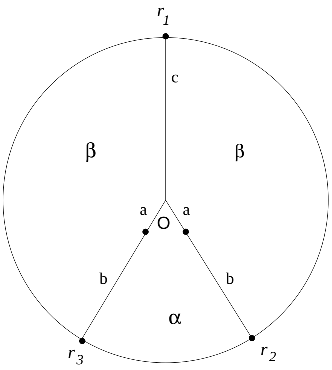
A radius ρ_i is of Type (of symmetry) 0 if there exists no radius ρ_j such that $W(\rho_i)^{\circlearrowleft} = W(\rho_j)^{\circlearrowright}$. Otherwise, ρ_i is said to be of Type 1.

A radius of Type t is said to be t-symmetric.

ρ_1 is 0-symmetric.

ρ_2 and ρ_3 are 1-symmetric.

Leader Election without Chirality



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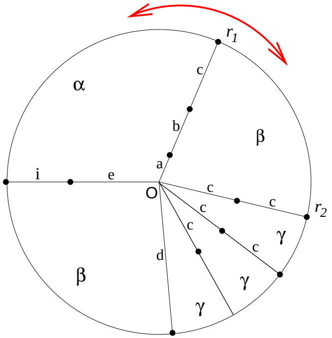
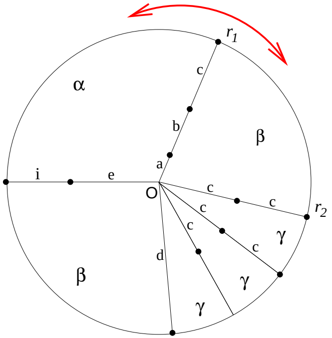


For each radius ρ_i, every robot computes $W(\rho_i)^{\circlearrowright}$ and $W(\rho_i)^{\circlearrowleft}$ of the form $(\mathit{type}, \mathit{radiusword}, \mathit{angle})$.

Leader Election without Chirality



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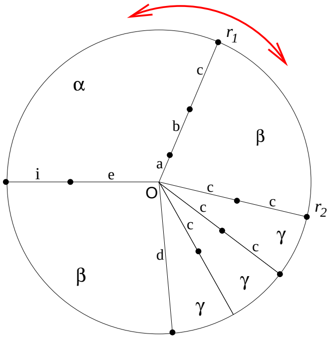
$$W(\rho_1)^{\circlearrowleft} = (0, abc, \alpha)(0, ei, \beta), (0, d, \gamma), (0, c, \gamma), (0, cc, \gamma), (0, cc, \beta)$$

Leader Election without Chirality



Lemma

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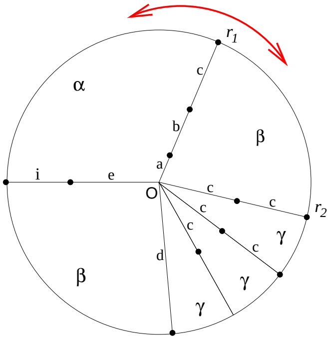
Lemma

If there exists a pair of radii $\{\rho_1, \rho_2\}$ so that $W(\rho_i)^{\circlearrowleft}$ or $W(\rho_i)^{\circlearrowright}$ is a Lyndon word ($i \in \{1,2\}$), then the robots are able to deterministically agree on the same leader if and only if ρ_1 and ρ_2 are 0-symmetric.

Leader Election without Chirality



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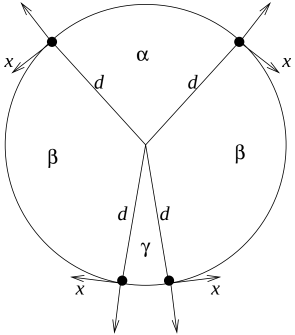

No leader exists.

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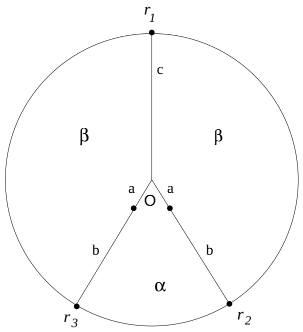
If there exists a pair of radii $\{\rho_1, \rho_2\}$ so that $W(\rho_i)^{\circlearrowright}$ or $W(\rho_i)^{\circlearrowleft}$ is a Lyndon word ($i \in \{1,2\}$), then the robots are able to deterministically agree on the same leader if and only if ρ_1 and ρ_2 are 0-symmetric.

Leader Election without Chirality



The robot on ρ_1 is the leader.

Lemma

If two distinct radii ρ_1 and ρ_2 exist such that both $W(\rho_1)$ and $W(\rho_2)$ are Lyndon words, then $\forall \rho$, $W(\rho) = (0, 0, 0)$.

Lemma

If there exists a pair of radii $\{\rho_1, \rho_2\}$ so that $W(\rho_i)^{\circlearrowright}$ or $W(\rho_i)^{\circlearrowleft}$ is a Lyndon word ($i \in \{1, 2\}$), then the robots are able to deterministically agree on the same leader if and only if ρ_1 and ρ_2 are 0-symmetric.

Leader Election without Chirality

Theorem

Assuming no chirality, a swarm of robots is able to deterministically agree on the same leader if and only if there exists a radius ρ such that $W(\rho)$ is a 0-symmetric Lyndon word.

Pattern Formation *vs.* Leader Election

Question

Given a swarm of n robots devoid of sense of direction, does the (arbitrary) pattern formation problem becomes solvable if the robots have the possibility to distinguish a unique leader?

Theorem

Assuming a cohort of $n \geq 4$ robots devoid of sense of direction, the pattern formation problem and the leader election problem are two equivalent problems, provided that the robots have the property of chirality.

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Proof of Equivalence

Lemma ($\Rightarrow$)

[Flocchini et al., 2008]
Assuming a cohort of $n \geq 3$ robots devoid of sense of direction, if it is possible to solve the pattern formation problem, then the leader election problem is solvable.

Lemma ($\Leftarrow$)

[Yamashita and Suzuki, 2008] [Petit and Dieudonné, 2009]
Assuming a cohort of $n \geq 4$ robots devoid of sense of direction, if it is possible to solve the leader election problem, then the pattern formation problem is solvable, provided that the robots have the property of chirality.

Proof of Equivalence

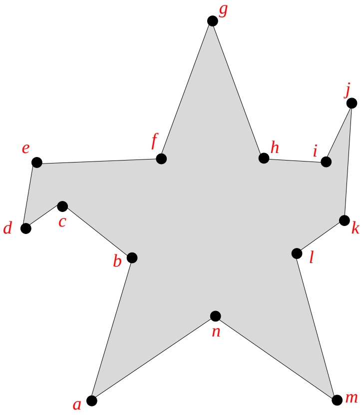
Lemma (⇒)

[Flocchini et al., 2008]
Assuming a cohort of $n \geq 3$ *robots devoid of sense of direction, if it is possible to solve the pattern formation problem, then the leader election problem is solvable.*

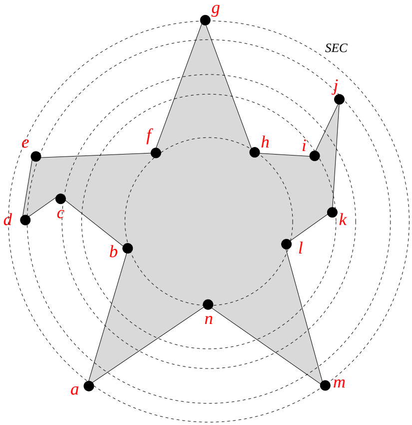
Lemma (⇐)

[Yamashita and Suzuki, 2008] [Petit and Dieudonné, 2009]
Assuming a cohort of $n \geq 4$ *robots devoid of sense of direction, if it is possible to solve the leader election problem, then the pattern formation problem is solvable, provided that the robots have the property of chirality.*

Algorithm



Algorithm



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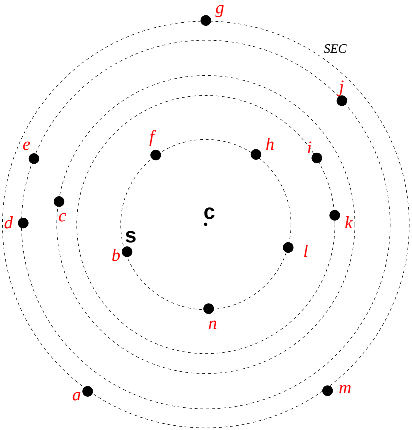

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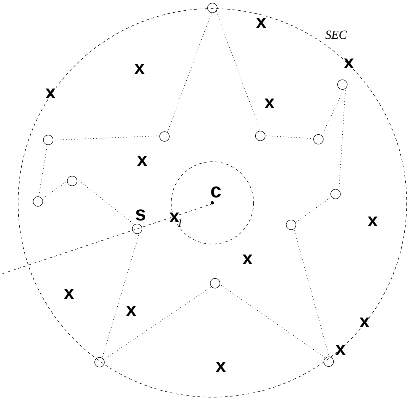

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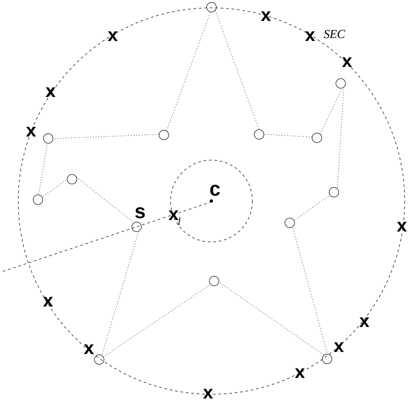

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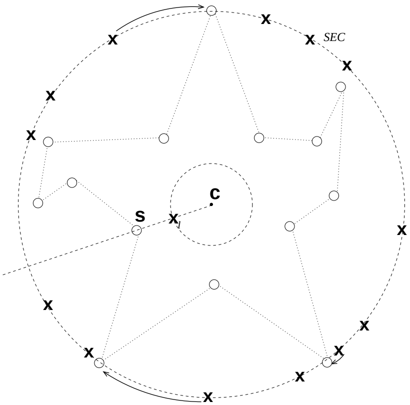

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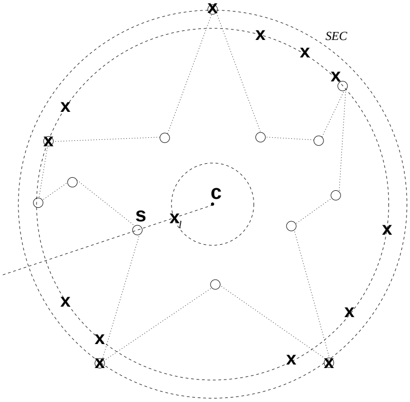

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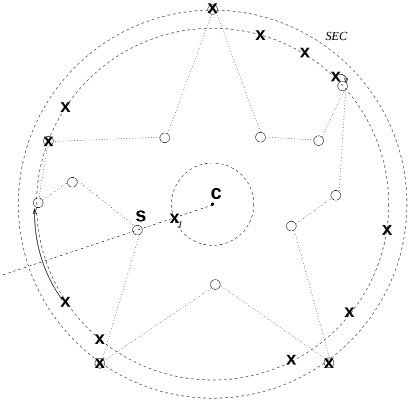

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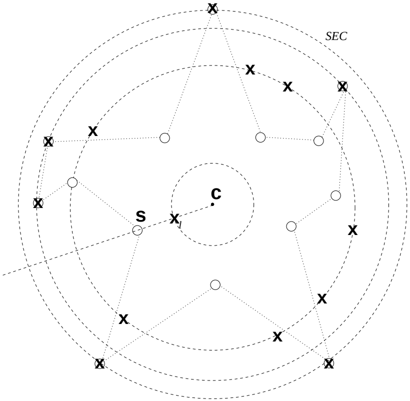

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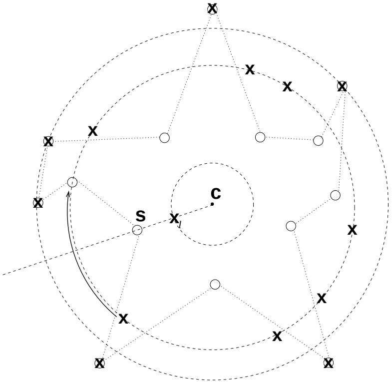

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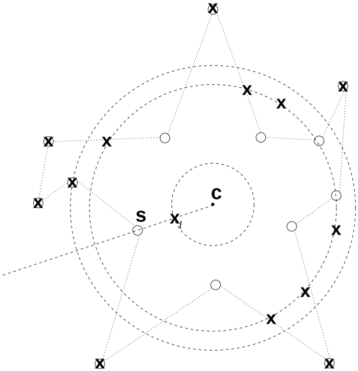

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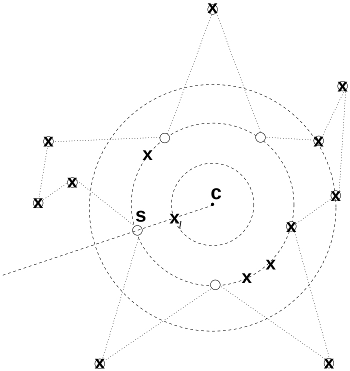

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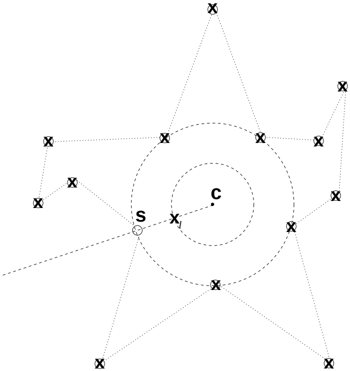

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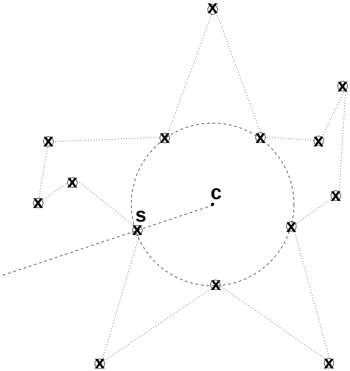

Pattern Formation *vs.* Leader Election

Remark

The equivalence also holds in CORDA. [Petit and Dieudonné, 2009]

Corollary

Assuming a cohort of $n \geq 4$ robots devoid of sense of direction in CORDA, the pattern formation problem and the leader election problem are two equivalent problems, provided that the robots have the property of chirality.

Open Question

Does the theorem still holds without chirality?

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Thank you.